3 Lagrangians & Hamiltonians

1. (Lecture Exercise) Consider the 1D action of the lectures

$$J[x(t)] = \int_{t_1}^{t_2} f(x, \dot{x}, t)dt$$

   Vary this action as explained in the lecture and obtain the corresponding Euler-Lagrange equations, assuming fixed boundary points.

2. (Lecture Exercise) Using that the distance along a curve $y(x)$ on a plane is given by

$$I[y(x)] = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

   (a) Vary this action integral and over the function $y(x)$ and obtain the Euler-Lagrande equations.

   (b) Solve the Euler-Lagrange equations, thus showing that the curve with the shortest distance between two points on a plane is a straight line.

3. (Lecture Exercise) By noting that the Lagrangian of a system can be seen as a function of momenta and positions and their corresponding derivatives using the definition of the Hamiltonian, i.e.

$$L(q_a, \dot{q}_a, p_a, \dot{p}_a) = -H(q_a, p_a, t) + \sum_a p_a \dot{q}_a$$

   (a) Look at the action integral again with this notation for the L dependence and convince yourself that the problem is now equivalent to a Lagrangian with 2-n coordinates $q_A = \{q_a, p_a\}$ and corresponding derivatives $\dot{q}_A = \{\dot{q}_a, \dot{p_a}\}$

   (b) Write the two sets of Euler Lagrange equations for each subset $\{q_a\}$ and $\{p_a\}$.

   (c) Replace the expression for the Lagrangian above in such equations and show that you obtain Hamilton's equations.

4. Solve again exercise 2 above, in the Hamiltonian formulation and contrast the two methods of solution.