2.1 Relativistic mechanics & systems of particles

1. (Lecture Exercise) Obtain $F^\mu \to (\gamma \vec{F}_N \cdot \vec{v}, \gamma \vec{F}_N)$, by boosting the small speed limit of the Newtonian force obtained in the lectures. Look at the time component and space components of the previous equation and show that

$$\begin{aligned} \frac{d}{dt}(\gamma m_0 \vec{v}) &= \vec{F}_N \\ \frac{d}{dt}(\gamma m_0) &= \vec{F}_N \cdot \vec{v} \end{aligned}$$

   where the second equation expresses the variation of the total energy $m = \gamma m_0$ ($E = mc^2 = \gamma m_0 c^2 = \gamma E_0$) of the particle by action of the force.

2. Consider a cosmic muon, which is formed through a collision of a cosmic ray at the top of the atmosphere. Knowing that the mass of the muon is $1.9.10^{-28} Kg$, that its total energy is $5GeV$, and that its decay time at rest it $T_0 = 2\mu s$ compute:

   (a) the magnitude of its 3-momentum

   (b) its velocity v/c and the space distance if it survives for a time T_0 at such speed.

   (c) the time T measured by an observer for which the muon is moving, until it decays (this is set in the proper rest frame of the muon) .

   (d) The distance it travelled during time T at speed v/c.

   (e) Comente on the two distances computed for times T_0 and T, and how it explains the observation of a large fraction of cosmic muons at the surface of the Earth.

3. Consider a process where an electron anihilates with a positron with spatial momenta along x, respectively $\vec{P} = (\pm p, 0, 0)$, (equal but opposite in sign) in a collision in the lab frame

   (a) Write the 4-momentum of each particle

   (b) Determine the conserved 4-momentum of the collision.

   (c) Consider the total 4-momentum of the last item and compute its invariant length $(P^\mu_{(tot)} P_{(tot)\mu})$.

   (d) Using the result of the last item, explain why it is not possible to anihilate just to one photon (hint: The photon is massless)

$$e^+ + e^- \to \gamma$$

   (e) Assuming the following anihilation process

$$e^+ + e^- \to \gamma + \gamma$$

   compute the 4-moomentum of each photon in the final state assuming they are emitted in opposite directions (as expected from the 3-momentum conservation)

4. Consider the decay of a (massive) pion into two massive particles (a muon and a neutrino)

$$\pi^+ \to \mu^+ + \nu_\mu$$

   Assuming that the pioin π^+ is at rest, use conservation of 4-momentum to compute the momentum of the muon and the neutrino, assuming they are emitted in opposite directions (as expected from the 3-momentum conservation).