1) Motivation

Particle Physics addresses the question:

What is matter made of at very short scales?

Remarkable fact → Everything built up from subatomic (sub-hadronic) particles which form bound states from the size scale of 0,001 times the proton size (0.88 x 10^{-15} m)

molecules ← atoms ← hadrons (proton, neutron, etc...)

↓

planets → galaxies → Universe: ~ 10^{27} m ⚠

Questions:

- What are the fundamental particles (building blocks)?
- How do they interact with each other?
- How to describe their properties and interactions mathematically?

→ Historically, particle physics knowledge built from:

1) Scattering experiments (smash things together)

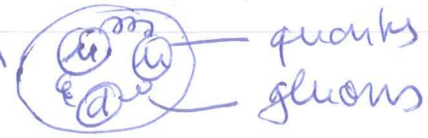
Ex: LHC proton + proton collisions → (7 TeV) ← (7 TeV)
4.5 TeV currently
(= 4500 x proton rest energy or mass) ⚠

2) Decays of unstable particles

Ex: LHC Higgs particle decays to two photons
$H \to \gamma\gamma$

3) Study of bound states

Ex: proton



1.1 The need for QFT

Two basic universal laws (independently of which particle we are studying): Quantum mechanics & Special Relativity

$k = 2\pi / \lambda$ ← de Broglie → Short distances

$\frac{v}{c}$ ↓ Fast

Classical Mechanics	Quantum Mechanics ($\hbar$)
Special Relativity (c)	Quantum Field Theory ($c, \hbar$)

→ Quantum field theory (QFT) is necessary to combine the two theories at high energies (short wavelengths) and high speed (close to c).

Examples:

In QM (Quantum mechanics), energy and momentum are quantized.

$$E = \hbar\omega \, , \quad p = \hbar k \Rightarrow \lambda = \frac{h}{p} \text{ (de Broglie wavelength)}$$

QM becomes important when $\lambda \sim$ size scale the object sees

a) Human being: Everyday size scale $\sim 1\,\text{m}$

Mass $\sim 75\,\text{kg}$

$v \sim 1\,\text{m}\,\text{s}^{-1}$

using $h = 6,626 \times 10^{-34}\,\text{J.s} \Rightarrow \lambda = \frac{h}{Mv} \sim 10^{-35}\,\text{m}$

⇒ everyday physics does not need QM

b) Thermal neutron at room temprature diffusing through a crystal lattice.

$m = M_{neutron} = 1.67 \times 10^{-27}$ kg

$E_{kinetic} = \frac{p^2}{2m}$ $\qquad E_{thermal} \simeq \frac{3}{2} k_B T$

If the kinetic energy of the neutron is provided by the thermally available energy per particle then

$$\frac{p^2}{2m} = \frac{3}{2} k_B T \Rightarrow p = \sqrt{3 k_B T m}$$

Then $\lambda \sim \frac{h}{\sqrt{3 k_B T m}} \sim 1.4 \times 10^{-10}$ m ~ interatomic distances in a crystal lattice.

⇒ QM important for (subatomic) particles in condensed matter systems.

→ What about special relativity?

a) Human being $\frac{v}{c} = \frac{1\,ms^{-1}}{3 \times 10^{9}\,ms^{-1}} \sim 10^{-9}$ (classical)

b) thermal neutrons $v = \frac{p}{m} = \sqrt{\frac{3 k_B T}{m}} \sim 2700\,ms^{-1}$

but $\frac{v}{c} \sim 10^{-6}$ (non-relativistic QM)

c) LHC experiments

proton + proton collisions $E_{pr.} \sim TeV = 10^{12} eV = 10^{12} \times 1.6 \times 10^{-19}$ J

Actually, in the relativistic limit $p = \frac{E}{c}$!

$\Rightarrow \lambda_B = \frac{h}{p} = \frac{hc}{E} \sim 1.2 \times 10^{-16}$ m → similar to proton charge radius 8×10^{-16} m

$$\Rightarrow \boxed{\begin{array}{l} \lambda \sim \text{Radius} \\ \frac{v}{c} \sim 1 \end{array}} \rightarrow$$

Highly relativistic & Quantum mechanical regime

$$\boxed{\Rightarrow \text{QFT} \ \triangle\!!}$$

Why Fields??

Fields will describe a "many particle theory".

In the relativistic regime, it is necessary since:

$E = mc^2$ allows energy converted to particle/anti-particle pairs, even if just for a short period allowed by uncertainty principle $\underline{\Delta E \Delta t \sim \hbar}$

$$\boxed{\Rightarrow \text{Particle number not conserved in QFT}}$$

1.2 Natural units

S.I. units are not very natural in particle physics

QFT $(c, \hbar)$

- v scale $\sim c$ ← speed of light
- $\hbar$ ← the other fundamental constant
- In particle physics GeV ~ proton rest energy/mass ($E_0 = m_0 c^2$)

$[c] = [L][T]^{-1}$

$[\hbar] = [M][L]^2[T]^{-1}$

⇒ 3 units. If we fix one, we can eliminate the others by setting $\hbar = c = 1$ /!\

$$c = 1 \quad \Rightarrow \quad [L] = [T]$$

$$\hbar = 1 \quad \Rightarrow \quad [M][L]^2 \cdot [T]^{-1} \Leftrightarrow [M][L] = 1$$

$$\Leftrightarrow [M] = [L]^{-1}$$

Possible choices:

Relativist fix $[L] = \text{meters (m)}$

$\Rightarrow T \to m$

$M \to m^{-1}$

(using $E = M\overset{=1}{c^2} \Leftrightarrow E = M$) $E \to m^{-1}$

Particle physics: Fix instead the energy scale to GeV

$$[\hbar] = [E][T] \Leftrightarrow 1 = [E][T] \Leftrightarrow [T] = [E]^{-1}$$

$\Rightarrow T \to \text{GeV}^{-1}$

$L \to \text{GeV}^{-1}$

$M \to \frac{\text{GeV}}{c^2} \to \text{GeV}$

Exercise: Consider a human with $v \sim 1\,\text{ms}^{-1}$. Using relativistic units, compute the time to travel 1 km @ this speed. Interpret the result in these units.

A first course on Quantum Field Theory
A summary of the SM

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1 Quantum field theory and particle physics

The discovery of special relativity, changed completely the way we describe interactions among particles. Instead of using one particle quantum mechanics, for consistency, in the relativistic limit one is forced to use field theory (for the moment you can think of field just as a function or a vector, with a value defined in the whole of space which evolves in time. In due course we will make this more rigorous.). A familiar case is electromagnetism which, at the classical level, is already described by a relativistic field.

In this picture, all the particles of a certain type are described by a field stretching out through the universe and determining the amount of particles of that type (for example the Maxwell field describes the photons permeating the Universe, and the Dirac field for the electrons). The job of particle physicists as been to identify the set of relativistic fields which describe all the fundamental particles seen in experiments. This has been largely achieved through high energy scattering experiments and the analysis of the decay of unstable particles, using the quantum field theory formulation of such fields.

1.1 Summary of the Standard Model

As explained above, the Standard Model of particle physics (SM) consists of a set of quantum relativistic fields which describes all physics down to length scales as small as 0.001 times the size of the proton. As far we know to the present date, all such particles are point like down to such scale, and all other particles (protons, neutrons, etc...) are bound states of SM particles. Below, we summarize the list of all the particles (or more rigorously , the set of fundamental fields) in the SM. Usually, they are separated into two groups: force carriers and matter particles.

Force carriers The 4 fundamental forces we know are described by bosonic fields. Surprisingly, only 3 forces are relevant in the domain of particle physics. The 3 are all described by vector fields, with some similarities with the Maxwell electromagnetic field. The fourth interaction, gravity, is curiously described by a more complicated field we will not address in this course.

The 3 interactions are:

- The electromagnetic field, which describes all electric and magnetic phenomenon down to the scale of electronic clouds in atoms. The corresponding particle we call the photon and label with the letter γ. The photon is a particle without mass (massless) which always travels at the speed of light and is electrically neutral. The photon is also neutral with respect to the strong nuclear force we will describe next. The magnitude of the interaction can be described by the fine structure constant, which is basically the coupling constant in the Coulomb force law between two electrons. Its value is $\alpha_{EM} \sim 0.007$. The potential of the electromagnetic force decays as $1/r$ which makes it a long range force.

- The weak nuclear force fields - This force describes the weak nuclear force, which is associated with phenomena such as neutron decay (and related nuclear decays), as well as various processes which occur in subatomic collisions at high energies. There are two particles associated with these fields: one is similar to the photon (electrically neutral) except that it is massive, and it is called the Z boson; the other is also massive and it is electrically charged with positive or negative charge $\pm e$ which we label $W^{\pm}$ respectively (they are particle/anti-particle). The weak force has a larger coupling constant $\alpha_W \sim 0.01$ however, due to the mass of the particles, it is short ranged ($V \sim e^{-\frac{m}{\hbar}r}/r$), which is why it only participates at sub-nuclear length scales.

- The strong nuclear force field - This describes the force which is responsible for the attraction of nucleons which keeps atomic nuclei bound. At a more fundamental level it is responsible for the structure of the proton and neutron (for example). That is, in the same way as an electrically neutral atom interacts with another neutral atom through forces of electromagnetic origin in nature (dipolar forces, London forces, etc...), for the interaction between nucleons, the strong nuclear force is a reflection of the fundamental strong force for keeping each nucleon bound.
  The fundamental particle/field responsible for such force is called the gluon, and it is designated by g_i. The gluon field has a special charge called "color charge", which is a more elaborated generalisation of electric charge. The index i represent 8 possible color charge configurations for the gluon. Next, we will see that some matter particle will have 3 possible color charge configurations.
  The magnitude of this force depends on the energy scale of the process. Surprisingly, its strength goes to zero at very short distances and increases at large distances. This is why it is necessary very high energy collisions to break the proton and study its structure.

- The fourth interaction is gravity. The associated hypothetical particle is called the graviton. However, its magnitude for the interaction of two particles (for example electrons) is $\sim 10^{-16}$, that is, it is completely irrelevant in the domain of particle physics.

In summary, the force carriers are described by the following fields

Particle	photon γ	Z boson	$W^{\pm}$ boson	g_i gluon	$g_{\mu\nu}$ graviton
electric charge	0	0	$\pm 1e$	0	0
color charge	0	0	0	8 possible	0
mass	0	$\sim 90\, m_{proton}$	$\sim 80\, m_{proton}$	0	0
range	long	short	short	long	long
coupling α	~ 0.007	~ 0.01	~ 0.01	~ 1	$\sim 10^{-16}$

Matter field The most familiar and relevant particle for everyday life is the electron e^-. It is a Fermion (obeys Fermi-Dirac statistics and has intrinsic spin 1/2), it has a negative electric charge, mass, and as far as we know, it is stable (it does not decay spontaneously). It does not have color charge, so it does not feel the strong force. Associated with the electron, it was found another neutral fermion, which participates in weak force interaction, which we call the electronic neutrino ν_e. Below we will see that neutrinos have a tiny mass, probably less than 0.001 times the electron mass. Surprisingly, in nature, there are two copies of each of these pairs of particles. For the electron we have the muon μ^- and the tau particle τ^-. The only difference is that they are increasingly more massive than the electron and are unstable. Each of these has an associated neutrino, that is we have a muon neutrino, ν_μ and a tau neutrino ν_τ. The masses of the neutrinos are not well known. Collectively, all of these particles are called leptons (from light), and all of them interact through the weak force.

Just like electrons are particles which interact with the electromagnetic force (or through the photon γ), there is another set of very important fermions which interact through the strong force (or the gluonsg_i). Those particles are called quarks and are the matter that make up the proton, neutron and other bound states generically called hadrons. Just as for leptons, there are 3 families with increasingly larger masses. The lightest is composed of the up quark u, which besides the color charge it has an electric charge $+\frac{2}{3}e$, and the down quark d with color charge and electric charge $-\frac{1}{3}$. The two other families are heavier replicas, the charm quark c and strange quark s (with same charges of the u e d respectively), and top t and bottom b (same corresponding charges). Due to the property that the strong force increases in magnitude with distance, quarks

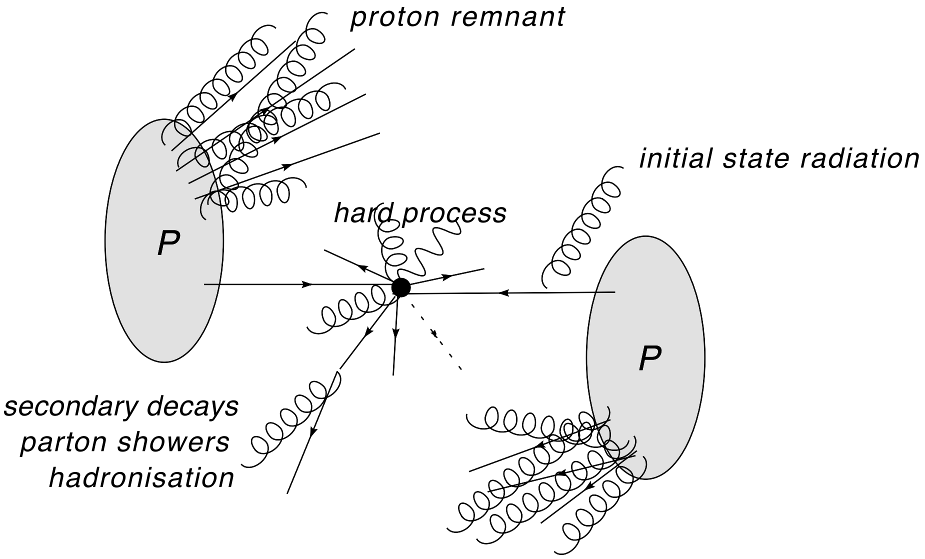

Figure 1: Schematics of the short range collision in a Proton, Proton collision at the LHC.

are not observed free at large distance, and are confined in the interior of hadron such as the proton. Only very high energy experiments allow to collide quarks at very short distances inside the proton and infer their properties as free particles (the strong force becomes zero at very short distances see Fig.1.).

To complete the theory of the SM, there is another particle, the Higgs boson h, recently discovered at the LHC, which is described by a massive scalar field. In fact, the Higgs boson has a central role in the theory and it can explain the mass generation mechanism for all massive particles in the SM. In summary, the matter observed experimentally in the SM is

Particle	e^-	ν_e	μ^-	ν_μ	τ^-	ν_τ
mass (m_p)	$5.5.10^{-4}$	$< 1.10^{-6}$	0.11	$< 1.10^{-6}$	1.9	$< 1.10^{-6}$
electric charge (e)	-1	0	-1	0	-1	0

Particle	u	d	c	s	t	b
mass (m_p)	~ 0.002	~ 0.005	~ 1.4	~ 0.1	~ 180	~ 4.5
electric charge (e)	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$

Most of these particles will not manifest themselves in daily life situations. On one hand, neutrinos are difficult to observe because they only interact through the weak force. Regarding leptons, the more massive ones decay to neutrinos and electrons. As for quarks, they are confined to bound states in hadrons, by gluons. In general, hadrons are divided in two groups: 1) barions, which can be seen as bound states of 3 quarks and 2) mesons which are bound states of quark and an anti-quark. In reality, these bounds states are more complicated than this simplistic view and are immersed in a color neutral sea of gluons and particle/anti-particle pairs.

For example, the proton can be seen as a bound state of two up quarks and a down quark

$$p \rightarrow uud \Rightarrow electric\ charge = +\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1 \tag{1}$$

and the neutron is a bound state of an up quark and a 2 down quark

$$n \rightarrow udd \Rightarrow electric\ charge = +\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0\,. \tag{2}$$